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Engineering Notes

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Analysis of Displaced Solar Sail Orbits with Passive Control

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Introduction

THE use of solar radiation pressure was first proposed by a Soviet pioneer of astronautics, Tsiolkovski, and the technology was greatly developed by NASA for a proposed comet Halley rendezvous mission in the 1970s [1,2]. Recently, many space applications of solar sails are proposed because solar sails enable some special missions which would be impossible for any conventional space propulsion. Such missions include displaced solar orbits, geocentric halo orbits, Mercury sun-synchronous polar orbit, artificial Lagrange points, and so on. Leipold and Wagner investigated the Mercury sun-synchronous polar orbit using solar sail propulsion to explore the inner solar system [3]. West investigated the new artificial Lagrange points created by solar sails to provide early warning of solar plasma storms before they reach the Earth [4]. McInnes and Simmons have done much work on the dynamics and control of solar sails on different exotic trajectories [5,6]. The stability of solar sails on displaced solar orbits with passive control is investigated in [7], and the results show that the sails are stable if the sail pitch angle is fixed with respect to a rotating frame. The passive stability can be realized by designing the configuration of the sail, which is investigated in [8]. Passive control is a good option for the solar sail because its large and complex structure may introduce some difficulties for active control.

In this Note, the global stability of the solar sail with passive control is investigated by considering the dynamics in an inertial frame. It is found that the sail is stable with any initial values, and the sail will oscillate in the vicinity of a nominal orbit that is uniquely determined by the angular momentum of the sail. The amplitudes of the oscillations are determined by the initial values of the radius and angular velocity.

Displaced Solar Orbit

An idealized solar sail is considered at position $\mathbf{r}$ in a reference frame rotating with angular velocity $\boldsymbol{\omega}$, as shown in Fig. 1. The displaced solar orbit is a circular orbit displaced above the ecliptic plane by directing a component of the solar radiation pressure force in a direction normal to the ecliptic. For a single plane sail, its orientation is defined by its normal vector $\mathbf{n}$, fixed in the rotating frame, and the sail performance is characterized by the sail lightness number β [5]. To keep an equilibrium in the rotating frame, the sail pitch angle α and sail lightness number β for a displaced sun-centered orbit of radius ρ, displacement z, and the orbital angular velocity ω should satisfy the relations [5]

$$\tan\alpha = \frac{(z/\rho)(\omega/\tilde{\omega})^2}{(z/\rho)^2 + 1 - (\omega/\tilde{\omega})^2} \tag{1}$$

$$\beta = \left[1 + \left(\frac{z}{\rho}\right)^2\right]^{\frac{1}{2}} \frac{\{(z/\rho)^2 + [1 - (\omega/\tilde{\omega})^2]^2\}^{\frac{3}{2}}}{[(z/\rho)^2 + 1 - (\omega/\tilde{\omega})^2]^2} \tag{2}$$

where $r^2 = \rho^2 + z^2$ and $\tilde{\omega}^2 = \mu/r^3$.

Passive control requires the pitch angle α to be fixed in the plane spanned by the angular vector $\boldsymbol{\omega}$ and the position vector $\mathbf{r}$. With passive control, displaced orbits of different sizes are stable when the linear variational equations are employed to analyze the stability [7]. However, if the differences between the positions of the sail and the reference displaced orbit are beyond the validity range of linear theory, the linear variational equations cannot be employed for stability analysis. This Note investigates the stability of a sail with any initial values.

Nominal Displaced Orbits Generated by Different Initial Values

As shown in Fig. 1, the frame $OXYZ$ is an inertial frame with the ecliptic plane as its OXY plane. A vector $\mathbf{r}$ in the inertial frame is described by three parameters, denoted as (ρ, θ, z), where ρ is the distance from the sun to the projection of the sail onto the ecliptic plane, θ is the angle between the X axis and the projection of $\mathbf{r}$ onto the ecliptic plane, and z is the distance from the sail to the ecliptic plane. The normal vector of the sail is assumed to be in the plane spanned by the vector $\mathbf{r}$ and the Z axis, and the pitch angle α is fixed in the same plane. Then, the dimensionless equations of motion can be described in the inertial frame as

$$\ddot{\rho} - \rho\dot{\theta}^2 = -\frac{1}{r^3}\rho + \beta\frac{1}{r^2}\cos^2\alpha\cos(\alpha + \varphi) \tag{3a}$$

$$\rho\ddot{\theta} + 2\dot{\rho}\,\dot{\theta} = 0 \tag{3b}$$

$$\ddot{z} = -\frac{1}{r^3}z + \beta\frac{1}{r^2}\cos^2\alpha\sin(\alpha + \varphi) \tag{3c}$$

where $r = \sqrt{\rho^2 + z^2}$ and $\varphi = \tan^{-1}(z/\rho)$.

The dimensionless equations are based on the reference length defined as the distance between the sun and the Earth, the reference mass defined as the mass of the sun, and the reference angular velocity defined as the mean angular velocity of the Earth rotating around the sun.

This set of three coupled ordinary differential equations can be reduced by integrating Eq. (3b)

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$$\dot{\theta} = \frac{\dot{\theta}_0 \rho_0^2}{\rho^2} \tag{4}$$

Equation (4) can be substituted into Eq. (3a) to obtain the decoupled equations of motion as

$$\ddot{\rho} = \frac{\dot{\theta}_0^2 \rho_0^4}{\rho^3} - \frac{1}{r^3}\rho + \beta\frac{1}{r^2}\cos^2\alpha\cos(\alpha+\varphi) \tag{5a}$$

$$\ddot{z} = -\frac{1}{r^3}z + \beta\frac{1}{r^2}\cos^2\alpha\sin(\alpha+\varphi) \tag{5b}$$

Three pairs of parameters are illustrated here to clarify the following discussions: the parameters of a reference orbit (ρ_r, z_r, ω_r), which are used to determine the values of α and β, the initial values of the sail (ρ_0, z_0, ω_0), and the parameters of the nominal orbit (ρ_n, z_n, ω_n), which are the equilibrium solutions to Eqs. (5a) and (5b). The parameters of the nominal orbit can be obtained by solving the algebraic equations given by

$$\frac{\dot{\theta}_0^2 \rho_0^4}{\rho^3} - \frac{1}{r^3}\rho + \beta\frac{1}{r^2}\cos^2\alpha\cos(\alpha+\varphi) = 0 \tag{6a}$$

$$-\frac{1}{r^3}z + \beta\frac{1}{r^2}\cos^2\alpha\sin(\alpha+\varphi) = 0 \tag{6b}$$

By substituting $\varphi = \tan^{-1}(z/\rho)$ into Eq. (6b), the ratio of the displacement to the radius can be obtained as

$$k = \frac{z_n}{\rho_n} = \frac{\beta\cos^2\alpha\sin\alpha}{1-\beta\cos^3\alpha}$$

Then, the parameters can be obtained by substituting the ratio into Eq. (6a), given by

$$\rho_n = \frac{\dot{\theta}_0^2\rho_0^4(1+k^2)^{3/2}}{1+\beta k\cos^2\alpha\sin\alpha - \beta\cos^3\alpha} \tag{7a}$$

$$z_n = k\rho_n \tag{7b}$$

The nominal orbit is determined by the initial angular momentum of the sail $c = \dot{\theta}_0\rho_0^2$, and Eq. (3b) shows that the angular momentum of the sail is conservative. Therefore, it can be concluded that the initial angular momentum is the only parameter that distinguishes different nominal orbits. Because the nominal orbit is the equilibrium position of the sail, sails with the same momentum will oscillate in the vicinity of the same nominal orbit. And the oscillation characteristics are dependent on the distribution of the initial angular momentum between the initial radius and initial angular velocity. The sail is expected to evolve on the reference orbit, and the necessary condition for the sail evolving on it is that the initial angular momentum of the sail is equal to that of the reference orbit, which guarantees that the reference orbit is the equilibrium position of the sail. If the necessary condition is violated, a nominal orbit different from the reference orbit will be generated. The stability of the nominal orbit is analyzed in the following section.

Stability of the Nominal Displaced Orbits

The nominal orbit is different from the reference orbit if the initial angular momentum of the sail is different from that of the reference orbit. Because the nominal orbit is the equilibrium position of the sail, the stability of the nominal orbit instead of the reference one is analyzed. The stability is discussed by applying a perturbation $\rho = \rho_n + \delta\rho$ and $z = z_n + \delta z$ to the decoupled equations to obtain the linear variational equations as

$$\begin{bmatrix}\delta\ddot{\rho}\\ \delta\ddot{r}\end{bmatrix} = \frac{1}{c^6}A(\alpha,\beta)\begin{bmatrix}\delta\rho\\ \delta r\end{bmatrix} \tag{8}$$

where $A(\alpha,\beta)$ is a constant matrix and $c^6 > 0$ is always true. Therefore, the variation of parameter c will change not stability but the undamped frequency of the system. All the nominal orbits generated by different initial angular momentum have the same stability. We have known that the reference orbit is one of the nominal orbits and it is stable with passive control, which has been validated in [7]. Therefore, all the nominal displaced orbits generated by different initial values are stable, which means that the solar sail will be always stable with passive control. If the sail is on the nominal orbit exactly at initial time, the sail will always be on this orbit. If the sail deviates from the nominal orbit, it will oscillate in the vicinity of this orbit with a frequency determined by Eq. (8). In addition, if the nominal orbit accords with the reference one, the variational equations that are linearized in the vicinity of the reference orbit can be used to investigate the stability of the sail [7]. The method proposed in this Note can be employed to analyze the stability if the nominal orbit is different from the reference one.

Period of the Nominal Displaced Orbit

The initial values change both the equilibrium solution to the decoupled equations and the period of the nominal orbit. To generate a displaced orbit with radius of ρ and displacement of z, Eqs. (1) and (2) should be satisfied. The period of the nominal orbit can be obtained by substituting the parameters (ρ_n, z_n, ω_n) and (ρ_r, z_r, ω_r) into these two equations, given by

$$\frac{T_n}{T_r} = \left(\frac{\rho_n}{\rho_r}\right)^{3/2} \tag{9}$$

The result shows that the square of the time of the orbital period is proportional to the cube of its average orbital radius, which is similar with Kepler's third law. The instant angular velocity of the sail will oscillate in the vicinity of the parameter ω_n, which may be different from the reference angular velocity ω_r. Therefore, the phase difference between the sail and reference position will increase as the time elapses. If the mission has a special requirement on the phase of the solar sail on the displaced orbit, the initial value of the sail should be chosen properly to satisfy the constraint $\dot{\theta}_0\rho_0^2 = \omega_r\rho_r^2$ or $\rho_n = \rho_r$.

Simulation Results

A displaced orbit of $\rho_r = 0.6$, $z_r = 0.4$, and $\omega_r = \omega_E$ (ω_E is the angular velocity of the Earth) is employed to investigate the responses of sails with different initial values. The reference orbit can be employed to generate the values of α and β. Once the solar sail is launched into space, the sail can oscillate in the vicinity of different nominal displaced orbits. These orbits are divided into subgroups according to the angular momentum and the parameters of these orbits generated by different initial values are illustrated in Figs. 2a–2c. The black hyperboloids denote the parameters of the orbits with

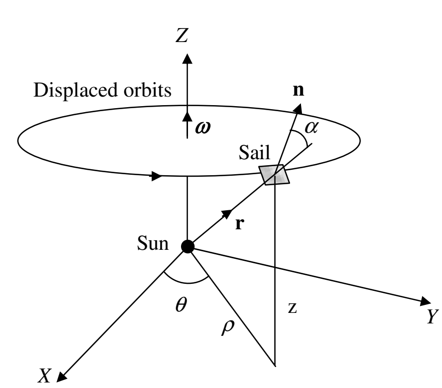
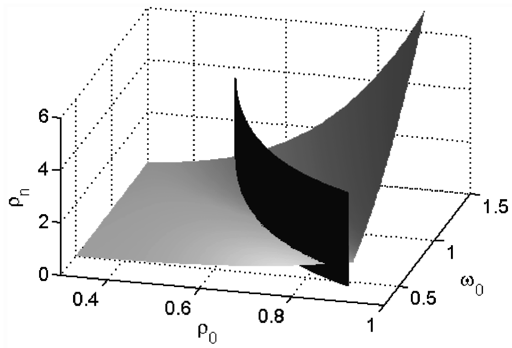
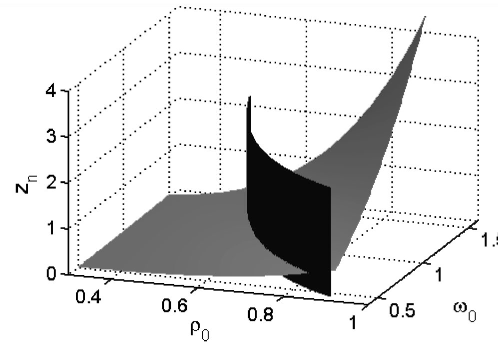


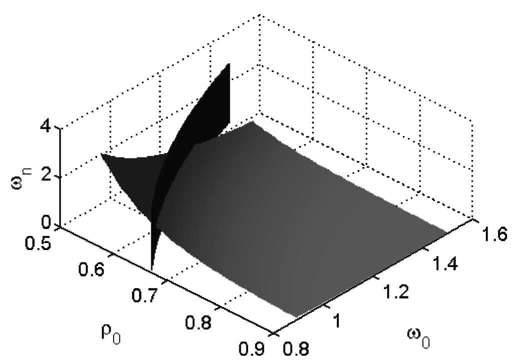
Fig. 1 Displaced solar orbit.



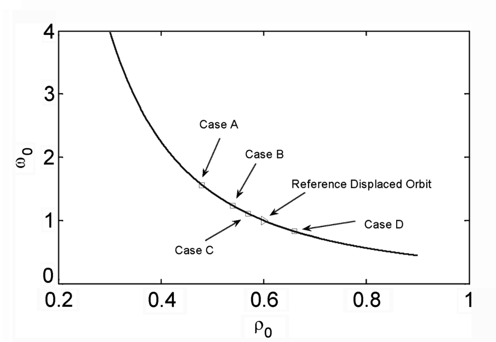
a) Radius of the nominal orbit



b) Displacement of the nominal orbit



c) Angular velocity of the nominal orbit



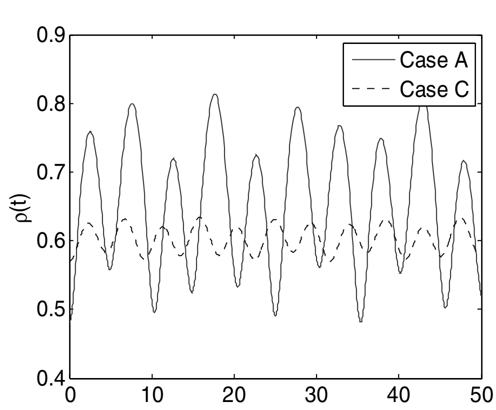
d) Different orbits expressed in the nominal space

Fig. 2 Parameters of nominal orbits and orbits expressed in new space.

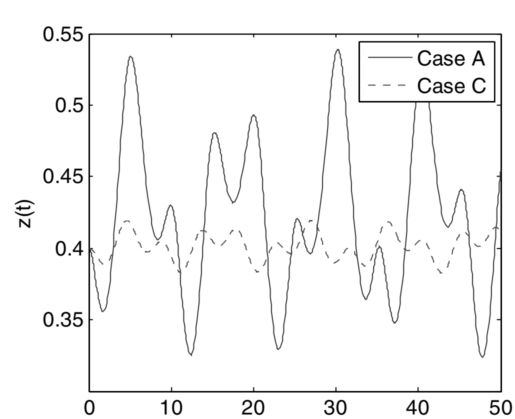
the same angular momentum, and the gray surfaces denote the orbital parameters with different initial radius and angular velocity. Sails with initial values on the intersection line of the two surfaces will oscillate in the vicinity of the same nominal orbit. In the space spanned by the initial radius and initial angular velocity, orbits with a fixed angular momentum form a hyperbola, as shown in Fig. 2d. A hyperbola maps the orbits with the angular momentum equal to that of a certain reference orbit, and the point on the hyperbola maps an oscillating orbit in the vicinity of the reference orbit. Four points are selected from the hyperbola to illustrate the characteristics of oscillations with different initial angular momentum distribution between the initial radius and angular velocity. The responses of the orbits for cases A and C are shown in Figs. 3a and 3b, respectively. The results show that the amplitude of oscillation increases with the distance between the two points, representing the oscillating orbit and reference orbit in Fig. 2d. The amplitude of the oscillation for the radius is almost equal to the difference between the initial radius ρ_0 and reference radius ρ_r. Therefore, to guarantee the sail close to the reference orbit, the initial angular momentum should be equal to that of the reference orbit, and the initial radius close to the reference radius.

Conclusions

The orbit of a solar sail with passive control will be always stable in the vicinity of a nominal displaced orbit that is determined by the initial angular momentum of the sail. Both the radius and displacement of the nominal displaced orbit increase with the initial angular momentum, and the period increases with the radius of the nominal orbit. In addition, the sail will oscillate in the vicinity of the nominal orbit, and the oscillation characteristics are determined by the initial radius and angular velocity. The amplitude of the oscillation increases with difference between the initial radius and radius of the nominal orbit.



a) Radius



b) Displacement

Fig. 3 Responses of the orbits with different initial values.

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